MISCELLANEOUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

16 Yale Senior Prize Problem.---Contributed by H. A. NEWTON, LL. D., Professor of Mathematics, Yale University, New Haven Connecticut.

The axes of two right cylinders whose bases are circles of 4 and 6 inches radius respectively, intersect at right angles. Compute to four decimal places the lengths of the *curves* of intersection of the two surfaces.

Solution by F. P. MATZ, D. Sc, Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.

Make $m=6$ inches, and $n=4$ inches; then the Cartesian equations of the cylinders become $z^2+x^2=m^2$ ....(1) and $y^2+x^2=n^2$ ....(2).

$$\therefore \frac{dz}{dx}=-\frac{x}{z}=-\frac{x}{\sqrt{(m^2-x^2)}} \quad \ldots.(3),$$

$$\text{and } \frac{dy}{dx}=-\frac{x}{y}=-\frac{x}{\sqrt{(n^2-x^2)}} \quad \ldots.(4).$$

Hence the expression for the *lengths* of the *curves* of intersection of the two surfaces becomes, *Todhunter's Integral Calculus*, p. 116,

$$L=8\int_0^n \sqrt{\left(1+\frac{x^2}{m^2-x^2}+\frac{x^2}{n^2-x^2}\right)}dx \ldots.(5).$$

Make $c^2=n^2/m^2$, and $x=n\sin\phi$; then $dx=n\cos\phi d\phi$. Transforming (5), etc.,

$$L=8\int_0^n \sqrt{\left(\frac{m^2n^2-x^4}{(m^2-x^2)(n^2-x^2)}\right)}dx=8n\int_0^{\frac{1}{2}\pi} \sqrt{\left(\frac{1-c^2\sin^4\phi}{1-c^2\sin^2\phi}\right)}d\phi$$

$$=8n\int_0^{\frac{1}{2}\pi}[1+\tfrac{1}{2}c^2\sin^2\phi+(\tfrac{3}{8}c^4-\tfrac{1}{2}c^2)\sin^4\phi+(\tfrac{5}{16}c^6-\tfrac{1}{4}c^4)\sin^6\phi+\text{etc.}]d\phi$$

$$=4\pi n\left[1+\frac{c^2}{16}+\frac{111c^4}{4016}+\text{etc.}\right]=51.9363+\text{inches.}$$

20. Proposed by SAMUEL HART WRIGHT, M. D., M. A., Ph. D., Penn Yan, Yates County, New York

When will the Dog-Star and the Sun *rise together* in lanitude $\lambda=+42°30'$, if the right Ascension of the said star be $\alpha=6h.\ 40m.\ 30s.$ and the Declination $\delta=-16°\ 33'\ 56''$?

Solution by F. P. MATZ, Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.

According to *Chauvenet's Spherical and Practical Astronomy*, Vol. I.,